

Multiple Layer Structure of Non-Abelian Vortex

Minoru Eto^{a,b}, Toshiaki Fujimori^c, Takayuki Nagashima^c,
Muneto Nitta^d, Keisuke Ohashi^e, and Norisuke Sakai^f

^a *INFN, Sezione di Pisa, Largo Pontecorvo, 3, Ed. C, 56127 Pisa, Italy*

^b *Department of Physics, University of Pisa Largo Pontecorvo, 3, Ed. C, 56127 Pisa, Italy*

^c *Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*

^d *Department of Physics, Keio University, Hiyoshi, Yokohama, Kanagawa 223-8521, Japan*

^e *Department of Applied Mathematics and Theoretical Physics,
University of Cambridge, CB3 0WA, UK*

^f *Department of Mathematics, Tokyo Woman's Christian University, Tokyo 167-8585, Japan*

Abstract

Bogomol'nyi-Prasad-Sommerfield (BPS) vortices in $U(N)$ gauge theories have two layers corresponding to non-Abelian and Abelian fluxes, whose widths depend nontrivially on the ratio of $U(1)$ and $SU(N)$ gauge couplings. We find numerically and analytically that the widths differ significantly from the Compton lengths of lightest massive particles with the appropriate quantum number.

⁰ e-mail addresses: minoru(at)df.unipi.it; fujimori,nagashi(at)th.phys.titech.ac.jp;
nitta(at)phys-h.keio.ac.jp; K.Ohashi(at)damtp.cam.ac.uk; sakai(at)lab.twcu.ac.jp.

1. *Introduction.* Many important properties of Abelian (ANO) vortex were found [1, 2, 3, 4, 5] since its discovery [6]. Recently vortices in $U(N)$ gauge theories (called non-Abelian vortices) were found [7, 8] and have attracted much attention [9] because they play an important role in a dual picture of quark confinement [8, 10] and are a candidate of cosmic strings [11] (see [12] for review report). The moduli space of $U(N)$ non-Abelian vortices was determined in [13] and study on interactions between non-BPS configurations started in [14]. Non-Abelian vortices in other gauge groups have been studied [15].

Although there have been much progress and wide applications, internal structures and dependence on gauge coupling constants have not yet been studied for (color) magnetic flux tubes. It is particularly important to study physical widths of vortices qualitatively and quantitatively, although it is not easy because no analytic solutions are known. It may be tempting to speculate that the width is determined by the Compton lengths of lightest massive particles with the appropriate quantum number. Purpose of this letter is to clarify intricate multiple layer structures of non-Abelian vortices by investigating numerically and analytically the equations of motion. Non-Abelian vortices have two distinct widths for $SU(N)$ and $U(1)$ fluxes. We clarify properties of these widths by making use of several approximations. It turns out that non-Abelian vortices are very different from ANO vortices and have much richer internal structures.

2. *Vortex equations and solutions.* Let us consider a $U(N)$ gauge theory with gauge fields W_μ for $SU(N)$ and w_μ for $U(1)$ and N Higgs fields H (N -by- N matrix) in the fundamental representation. We consider the Lagrangian $\mathcal{L} = K - V$ which can be embedded into supersymmetric theory with eight supercharges

$$K = \text{Tr} \left[-\frac{1}{2g^2} (F_{\mu\nu})^2 + \mathcal{D}_\mu H \mathcal{D}^\mu H^\dagger \right] - \frac{1}{4e^2} (f_{\mu\nu})^2, \quad (1)$$

$$V = \frac{g^2}{4} \text{Tr} [\langle HH^\dagger \rangle^2] + \frac{e^2}{2} (\text{Tr} [HH^\dagger - c\mathbf{1}_N])^2, \quad (2)$$

where $\langle X \rangle$ stands for a traceless part of a square matrix X . Our notation is $\mathcal{D}_\mu H = (\partial_\mu + iW_\mu + iw_\mu \mathbf{1}_N)H$, $F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + i[W_\mu, W_\nu]$ and $f_{\mu\nu} = \partial_\mu w_\nu - \partial_\nu w_\mu$. We have three couplings: $SU(N)$ gauge coupling g , $U(1)$ gauge coupling e and Fayet-Iliopoulos parameter $c > 0$.

The Higgs vacuum $H = \sqrt{c} \mathbf{1}_N$ is unique and is in a color-flavor $SU(N)_{\text{C+F}}$ locking phase. Mass spectrum is classified according to representations of $SU(N)_{\text{C+F}}$ as $m_g \equiv g\sqrt{c}$ for non-Abelian fields $\phi_N = (W, \langle H \rangle)$ and $m_e \equiv e\sqrt{2Nc}$ for Abelian fields $\phi_A = (w, \text{Tr}(H - \sqrt{c}\mathbf{1}_N))$. The non-Hermitian part of H is eaten by the $U(N)$ gauge fields. A special case of $m_g = m_e$ [7] has

been mostly considered so far, which is equivalently

$$\gamma = 1 \quad \text{with} \quad \gamma \equiv \frac{g}{e\sqrt{2N}} = \frac{m_g}{m_e}, \quad (3)$$

but we study general cases in this Letter.

Let us consider static vortex-string solutions along x_3 -axis. The BPS equations for the non-Abelian vortex are

$$\bar{\mathcal{D}}H = 0, \quad \frac{F_{12}}{m_g^2} = \frac{\langle HH^\dagger \rangle}{2c}, \quad \frac{f_{12}}{m_e^2} = \frac{\text{Tr}(HH^\dagger - c\mathbf{1}_N)}{2c}, \quad (4)$$

with $\bar{\mathcal{D}} = (\mathcal{D}_1 + i\mathcal{D}_2)/2$. The tension of k -vortex is $T_k = -c \int d^2x \text{Tr}[f_{12}\mathbf{1}_N] = 2\pi kc$. No analytic solutions have been known whereas a numerical solution was first found in [8]. For $k = 1$ vortex, we take $H = S^{-1}H_0$, $\bar{W} = -iS^{-1}\bar{\partial}S$ [$2\bar{W} \equiv (W_1 + w_1\mathbf{1}_N) + i(W_2 + w_2\mathbf{1}_N)$] with diagonal matrices $H_0 = \text{diag}(re^{i\theta}, 1, \dots, 1)$ and $cSS^\dagger = e^{(\psi_e + \frac{1}{N}\log r^2)\mathbf{1}_N + (\psi_g + \frac{N-1}{N}\log r^2)T}$. Here, $T \equiv \text{diag}(1, -\frac{1}{N-1}, \dots, -\frac{1}{N-1})$ and ψ_e and ψ_g are real functions of the radius $r > 0$ of the polar coordinates (r, θ) in x_1, x_2 plane. Then we get

$$\frac{\Delta\psi_e}{m_e^2} + \frac{1}{N}e^{-\psi_e} \left(e^{-\psi_g} + (N-1)e^{\frac{\psi_g}{N-1}} \right) = 1, \quad (5)$$

$$\frac{\Delta\psi_g}{m_g^2} + \frac{N-1}{N}e^{-\psi_e} \left(e^{-\psi_g} - e^{\frac{\psi_g}{N-1}} \right) = 0, \quad (6)$$

with $\Delta f(r) = \partial_r(r\partial_r f(r))/r$. The boundary conditions are $\psi_e, \psi_g \rightarrow 0$ ($r \rightarrow \infty$) and $N\psi_e, \frac{N}{N-1}\psi_g \rightarrow -\log r^2$ ($r \rightarrow 0$). The fluxes and the Higgs fields are expressed by

$$f_{12} = -\frac{1}{2}\Delta\psi_e, \quad F_{12} = -\frac{1}{2}\Delta\psi_g T, \quad H = \sqrt{c} \text{diag}(h_1, h_2, \dots, h_2) \quad (7)$$

with $h_1 = e^{-\frac{\psi_e + \psi_g}{2} + i\theta}$ and $h_2 = e^{-\frac{1}{2}(\psi_e - \frac{\psi_g}{N-1})}$. The amount of the Abelian flux is $1/N$ and the non-Abelian flux is $(N-1)/N$ of the ANO vortex.

We found the following theorems for Eqs. (5) and (6)

- a) $\psi_{e,g} > 0$, $\partial_r \psi_{e,g} < 0$ and $\Delta\psi_{e,g} > 0$
- b) $|h_1| < |h_2|$, $|h_1| < 1$ and $\partial_r |h_1| > 0$
- c) $\partial_r |h_2| \gtrless 0$ and $1 \gtrless h_2 \gtrless \sqrt{N/(N + \gamma^2 - 1)}$ for $\gamma \gtrless 1$

All these can be proved by using the following theorem for an analytic function $f(r)$ satisfying $f(r) < 0 \Rightarrow \Delta f(r) < 0$: If $\partial_r f(0) \leq 0$ and $f(\infty) = 0$, then $f(r) \geq 0$ for $\forall r \in (0, \infty)$. In the case of

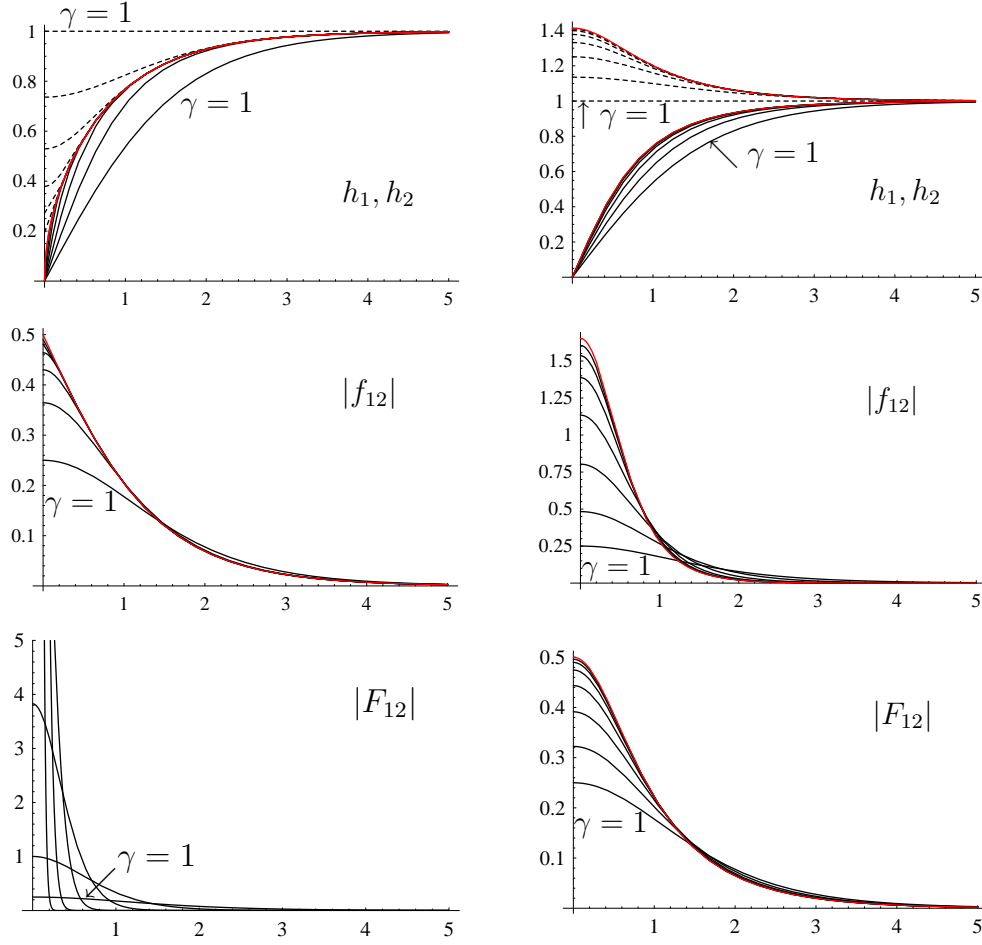


Fig. 1: $h_{1,2}$ (solid, broken lines) and $B_{e,g}$. The left panels ($m_e = 1$) for $\log \gamma = 0, 1, 2, 3, 4, 5$ and ∞ . The right panels ($m_g = 1$) for $\log \gamma = 0, -0.5, -1, \dots, -3$ and $-\infty$.

$\gamma = 1$, we get $N\psi_e = \frac{N}{N-1}\psi_g \equiv \psi_{\text{ANO}}$ and the above equations reduce to $\Delta\psi_{\text{ANO}} = m_e^2(1 - e^{-\psi_{\text{ANO}}})$ with boundary condition $\psi_{\text{ANO}} \rightarrow 0$ ($r \rightarrow \infty$) and $\psi_{\text{ANO}} \rightarrow -\log r^2$ ($r \rightarrow 0$).

Numerical solutions for $N = 2$ for a wide range of γ (including $\gamma = 0, \infty$) are shown in Fig. 1. Winding field h_1 is not sensitive on γ while unwound field h_2 is. As m_g being sent to ∞ ($\gamma \rightarrow \infty$), the non-Abelian flux F_{12} becomes very sharp and finally gets to singular. Interestingly, the Abelian flux f_{12} is kept finite there. In a region $\gamma < 1$ ($m_e > m_g$), on the other hand, the Abelian flux is a bit smaller than the non-Abelian tube. Surprisingly, the fluxes remain finite even in $m_e \rightarrow \infty$ limit.

3. Asymptotic width. Let us investigate the vortex solution by expanding (5) and (6) in region $r \gg \max\{m_e^{-1}, m_g^{-1}\}$ where $|\psi_e|, |\psi_g| \ll 1$. We keep only the lowest-order term in ψ_e while

keeping terms up to next to leading order in ψ_g in Eq.(5):

$$(\Delta - m_e^2)\psi_e + \frac{m_e^2\psi_g^2}{2(N-1)} = 0, \quad (\Delta - m_g^2)\psi_g = 0. \quad (8)$$

The solution is given by the second modified Bessel function $K_0(r)$, and approximated as

$$\psi_e \simeq \begin{cases} c_e \sqrt{\frac{\pi}{2m_e r}} e^{-m_e r}, \\ \frac{\pi c_g^2}{4(N-1)(1-4\gamma^2)} \frac{e^{-2m_g r}}{m_g r}, \end{cases} \quad \psi_g \simeq c_g \sqrt{\frac{\pi}{2m_g r}} e^{-m_g r}, \quad (9)$$

with $c_{e,g}$ being dimensionless constants, see Table 1. The asymptotic behavior of ψ_e changes at

γ	c_e	c_g	b_e	b_g	a_γ
0	—	1.1363	0	0.75905	$\sqrt{2}$
0.25	—	1.1853	0.31719	0.73163	1.31688
0.5	—	1.3090	0.47907	0.68393	1.18361
0.75	2.196	1.4852	0.55921	0.64006	1.07932
1	1.70786	1.70786	0.60329	0.60329	1
1.5	1.4715	2.3031	0.64726	0.54697	0.88820
2	1.4037	3.15	0.66773	0.50604	0.81226
2.5	1.3746	4.32	0.67897	0.47469	0.75640
3	1.3594	6.0	0.68584	0.44969	0.71301
∞	1.3267	—	0.70653	0	0

Table 1: Numerical data for $k = 1$ $U(2)$ vortex.

$\gamma = 1/2$ (upper for $\gamma \geq 1/2$ and lower $\gamma \leq 1/2$). Similar phenomenon was observed for the non-BPS ANO vortex [3, 4]. The origin of $\psi_e(\psi_g)$ is (non-)Abelian fields $\phi_A(\phi_N)$ with mass $m_e(m_g)$, and the $\gamma = 1/2$ threshold can be interpreted as follows. The expansion of the Lagrangian with respect to small $\phi_{A,N}$ contains the triple couplings $\phi_A\phi_N^2$. For $m_e \leq 2m_g$, asymptotics for $\phi_{A,N}$ are given by $K_0(m_{e,g}r)$ as the two-dimensional Green's function. When $m_e > 2m_g$, the particles ϕ_A decay into two particles ϕ_N^2 through these couplings, and thus, ϕ_A exhibits the asymptotic behavior $e^{-2m_g r}$ below $\gamma = 1/2$ like Eq. (9). On the contrary, even for $\gamma > 2$, ϕ_N does not behave as $e^{-2m_e r}$ since there is no triple coupling $\phi_N\phi_A^2$ due to the traceless condition for ϕ_N .

Let us define *asymptotic width of the vortex* by an inverse of the decay constant in Eq.(9):

$$L_e = \begin{cases} 2/m_e & \text{for } \gamma \geq 1/2, \\ 2/(2m_g) & \text{for } \gamma < 1/2, \end{cases} \quad L_g = 2/m_g. \quad (10)$$

Here the factor 2 is put in the numerator to match with another definition in Eq.(14). The asymptotic width of Abelian vortex is bigger than the non-Abelian one when $\gamma > 1$ and vice versa for $1/2 \leq \gamma < 1$. For $\gamma = 1$, the two widths are the same. The case $\gamma < 1/2$ indicates a significant modification, where the Abelian flux tube is supported by the non-Abelian flux tube. This answers the question why the Abelian vortex does not collapse even in the $m_e \rightarrow \infty$ limit. When $\gamma \gg 1$, the thin non-Abelian flux hidden by fat Abelian flux cannot be correctly measured by L_g . We now turn to another definition of vortex width which reflect the size of the vortex core more faithfully.

4. *Core widths.* Let us consider a region near the vortex core. We expand fields by

$$\psi_e \approx -\frac{2}{N} \log b_e m_e r, \quad \psi_g \approx -2 \frac{N-1}{N} \log b_g m_g r. \quad (11)$$

Dimensionless constants $b_{e,g}$ are related to $h_2(r=0)$ by

$$a_\gamma \equiv h_2(0) = (b_e/\gamma b_g)^{1/N}. \quad (12)$$

See Table 1. These are important since they are related to the maximum values of the magnetic fluxes at $r=0$

$$B_e = -\frac{m_e^2}{2} \left(1 - \frac{N-1}{N} a_\gamma^2 \right), \quad B_g = -\frac{m_g^2}{2} \frac{N-1}{N} a_\gamma^2. \quad (13)$$

Widths of the magnetic fluxes can be estimated by using a step function $\Theta(x)$ as $F_{12} = B_g \Theta(\tilde{L}_g - r)T$ and $f_{12} = B_e \Theta(\tilde{L}_e - r)$ keeping amounts of the fluxes as $|B_e| \times \pi \tilde{L}_e^2 = 2\pi/N$ and $|B_g| \times \pi \tilde{L}_g^2 = 2(N-1)\pi/N$:

$$\tilde{L}_e = \frac{2}{m_e \sqrt{N - (N-1)a_\gamma^2}}, \quad \tilde{L}_g = \frac{2}{m_g a_\gamma}, \quad (14)$$

We call \tilde{L}_e and \tilde{L}_g as the *core widths* of the vortex. In the case of $\gamma = 1$, \tilde{L}_e and \tilde{L}_g coincide because of $a_{\gamma=1} = h_2 = 1$. In Fig. 2, we show the core widths numerically in the case of $N = 2$, which are analytically reinforced as we will discuss. We again observe that the Abelian core does not collapse even when $m_e \gg 1$ ($\gamma \ll 1$).

Mass dependence of the core widths coincides with one of the asymptotic widths $L_{e,g}$ given in Eq. (10), except for $\tilde{L}_g(\gamma > 1)$, see Fig. 2. The asymptotic width L_g is independent of m_e whereas the core width \tilde{L}_g depends on m_e . This is because $\tilde{L}_{e,g}$ more faithfully reflects the multilayer

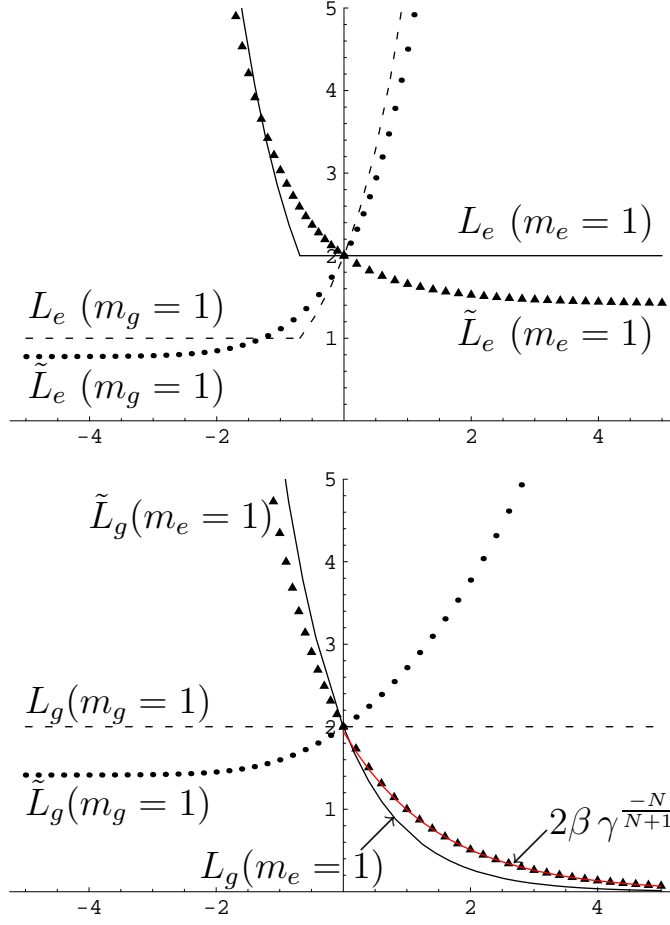


Fig. 2: The core and asymptotic widths v.s $\log \gamma$.

structure in the large intermediate region of r for the strong coupling regime ($\gamma \gg 1$), to which we now turn.

5. Two strong coupling limits. Here we study two limits: i) $m_g \rightarrow \infty$ with m_e fixed, and ii) $m_e \rightarrow \infty$ with m_g fixed. In the former limit ($\gamma \rightarrow \infty$ with m_e fixed), all the fields with mass m_g become infinitely heavy and integrated out from the theory. As a result, the original $U(N)$ gauge theory becomes the Abelian theory with one $SU(N)$ singlet field $B \equiv \det H$. Note that the target space is \mathbb{C}/\mathbb{Z}_N because $U(1)$ charge of B is N . Eq. (6) is solved by $\psi_{g,\infty} = 0$ while $\psi_{e,\infty}$ is determined by Eq. (5)

$$\frac{\Delta \psi_{e,\infty}}{m_e^2} = 1 - e^{-\psi_{e,\infty}}, \quad \psi_{e,\infty} \xrightarrow{r \rightarrow 0} -\frac{2}{N} \log m_e b_{e,\infty} r \quad (15)$$

where suffix ∞ denotes $\gamma = \infty$: $\psi_{g,\infty} \equiv \psi_g|_{\gamma \rightarrow \infty}$. The boundary condition tells that vorticity is fractional $k = 1/N$. This way the non-Abelian flux tube collapses and the $U(N)$ non-Abelian vortex reduces to the $1/N$ fractional Abelian vortex. This solution helps us to understand the

non-Abelian vortex for $\gamma \gg 1$ better. Since $e^{-\psi_e} \approx (m_e b_{e,\infty} r)^{2/N}$ ($r \ll 1/m_e$) for $\gamma \gg 1$, ψ_g for $r \ll 1/m_e$ is well approximated by a solution of the following

$$\frac{\Delta\psi_g}{\tilde{m}_g^2} + \frac{N-1}{N}(\tilde{m}_g r)^{\frac{2}{N}} \left(e^{-\psi_g} - e^{\frac{\psi_g}{N-1}} \right) = 0, \quad (16)$$

where the parameter $\tilde{m}_g \equiv m_g(b_{e,\infty}\gamma^{-1})^{\frac{1}{N+1}}$ has a mass dimension. Therefore ψ_g has asymptotic behavior in the middle region $1/\tilde{m}_g \ll r \ll 1/m_e$

$$\psi_g \approx \tilde{c}_g K_0 \left(\frac{N}{N+1} (\tilde{m}_g r)^{\frac{N+1}{N}} \right) \ll 1, \quad (17)$$

and $\psi_g \approx -\frac{2(N-1)}{N} \log(\tilde{b}_g \tilde{m}_g r)$ for $r \ll 1/\tilde{m}_g$. Here \tilde{b}_g, \tilde{c}_g are determined numerically and independent of γ , for instance, $\tilde{b}_g = 0.74672$, $\tilde{c}_g = 0.63662$ for $N = 2$. Comparing this with Eq. (11), we find $b_g \approx \tilde{b}_g [b_{e,\infty}\gamma^{-1}]^{\frac{1}{N+1}}$ and $a_\gamma \approx \tilde{b}_g^{-\frac{1}{N}} [b_{e,\infty}\gamma^{-1}]^{\frac{1}{N+1}}$ for $\gamma \gg 1$.

In the second limit ($\gamma \rightarrow 0$ with m_g fixed), all the fields with the mass m_e are integrated out. The model reduces to a $\mathbb{C}P^{N^2-1}$ model with $SU(N)$ isometry [in $SU(N^2-1)$] gauged. Eq. (5) is solved by $e^{\psi_{e,0}} = (e^{-\psi_{g,0}} + (N-1)e^{\frac{\psi_{g,0}}{N-1}})/N$ while $\psi_{g,0}$ is determined by

$$\Delta\psi_{g,0} = m_g^2 \frac{(N-1) \left(1 - e^{-\frac{N}{N-1}\psi_{g,0}} \right)}{(N-1) + e^{-\frac{N}{N-1}\psi_{g,0}}}, \quad (18)$$

where the suffix 0 denotes $\gamma \rightarrow 0$: $\psi_{g,0} \equiv \psi_g|_{\gamma \rightarrow 0}$. This is a *new* σ -model lump with the non-Abelian flux accompanied with the internal orientation $\mathbb{C}P^{N-1}$. Again we can make use of this solution to understand the non-Abelian vortex for $\gamma \ll 1$. Let us define $\alpha^2 \equiv \frac{\Delta\psi_{g,0}(0)}{\Delta\psi_{e,0}(0)} = \frac{B_g}{B_e}|_{\gamma \rightarrow 0}$ which turns out to be finite $\alpha^2 = (N-1)/(1+4b_g^2|_{\gamma \rightarrow 0})$. Since $\Delta\psi_{g,0}(0) = m_g^2$ and $\Delta\psi_{e,0}(0) = \lim_{\gamma \rightarrow 0} m_g^2 \gamma^{-2} (1 - (N-1)a_\gamma^2/N)$, we find $a_\gamma = a_0 (1 - \gamma^2/(2\alpha^2) + \dots)$, $a_0 = \sqrt{N/(N-1)}$ for $\gamma \ll 1$.

6. Summary and Discussion. We have proposed two length scales for fluxes of non-Abelian vortices: asymptotic widths $L_{e,g}$ in Eq. (10) and core widths $\tilde{L}_{e,g}$ in Eq. (14). By using the asymptotics of a_γ obtained above, the core width is summarized as

$$\left\{ \tilde{L}_e, \tilde{L}_g \right\} \simeq \begin{cases} \left\{ \frac{2\alpha}{m_g \sqrt{N}}, \frac{2}{m_g} \sqrt{\frac{N-1}{N}} \right\} & (\gamma \ll 1), \\ \left\{ \frac{2}{m_e \sqrt{N}}, \frac{2\beta}{m_g} \left(\frac{m_g}{m_e} \right)^{\frac{1}{N+1}} \right\} & (\gamma \gg 1), \end{cases} \quad (19)$$

where α and β depend only on N and are determined numerically, for instance $\alpha = 0.55010, \beta = 0.97022$ for $N = 2$. The core and asymptotic widths have the same mass dependence except

for \tilde{L}_g and L_g for $\gamma \gg 1$. Interestingly, the Abelian flux does not collapse even when $m_e \rightarrow \infty$ ($\gamma \ll 1$). For $\gamma \gg 1$, the thin non-Abelian flux is hidden by fat Abelian flux, so that the true width cannot be captured by the asymptotics at $r \gg m_e^{-1}$. Instead we should use improved approximation given in Eq. (17) to measure the non-Abelian flux. Indeed, the decay constant in Eq. (17) is \tilde{m}_g^{-1} whose mass dependence is the same as one of \tilde{L}_g for $\gamma \gg 1$.

In the limit $m_g \rightarrow 0$, the original $U(N)$ gauge theory reduces to $U(1)$ gauge theory coupled to N^2 Higgs fields. Eq. (19) tells us that the vortex is diluted and vanishes in this limit. This is consistent with the fact that there is no (smooth) vortex solution with a winding number $1/N$ in that $U(1)$ theory. The minimal vortex in the $U(1)$ theory corresponds to N vortices in the original theory. The dilution is expected to be avoided and all the fields with mass of the order of m_g decouple, when N vortices are arranged as $H = f(r)\mathbf{1}_N$. In the limit $m_e \rightarrow 0$, there is no BPS vortex solution since the $U(1)$ gauge field is decoupled from the Higgs fields. Actually, according to Eq. (19), one can find both of the Abelian and the non-Abelian fluxes are diluted again even in this limit due to the factor $\gamma^{\frac{1}{N+1}}$. Monopoles/instantons attached by vortices are known to exist [10]. The above observation implies that such configurations reduce to a monopole/instanton configurations in the $SU(N)$ gauge theory in that limit, and strongly supports the correspondence between the moduli spaces of them.

It is interesting to study relation between non-BPS vortices and monopoles. It was found that monopoles do not collapse when the Higgs mass is very large [16].

M.E. and K.O. would like to thank S.B.Gudnason, N.Manton, D.Tong and W.Vinci for useful discussions. This work is supported in part by Grant-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science and Technology, Japan No.17540237, No.18204024 (N.S.) and No.20740141 (M.N.). The work is also supported by the Research Fellowships of the Japan Society for the Promotion of Science for Research Abroad (M.E. and K.O.) and for Young Scientists (T.F. and T.N.).

References

- [1] H. J. de Vega and F. A. Schaposnik, Phys. Rev. D **14**, 1100 (1976).
- [2] C. H. Taubes, Commun. Math. Phys. **72**, 277 (1980).

- [3] B. Plohr, J. Math. Phys. **22** (1981) 2184.
- [4] L. Perivolaropoulos, Phys. Rev. D **48**, 5961 (1993)
- [5] J. M. Speight, Phys. Rev. D **55**, 3830 (1997)
- [6] H. B. Nielsen and P. Olesen, Nucl. Phys. B **61**, 45 (1973).
- [7] A. Hanany and D. Tong, JHEP **0307**, 037 (2003).
- [8] R. Auzzi, S. Bolognesi, J. Evslin, K. Konishi and A. Yung, Nucl. Phys. B **673**, 187 (2003).
- [9] Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, Phys. Rev. D **71**, 065018 (2005); M. Eto, M. Nitta and N. Sakai, Nucl. Phys. B **701**, 247 (2004); A. Gorsky, M. Shifman and A. Yung, Phys. Rev. D **71**, 045010 (2005); M. Shifman and A. Yung, Phys. Rev. D **72**, 085017 (2005); M. Eto, K. Konishi, G. Marmorini, M. Nitta, K. Ohashi, W. Vinci and N. Yokoi, Phys. Rev. D **74**, 065021 (2006); M. Edalati and D. Tong, JHEP **0705**, 005 (2007); D. Tong, JHEP **0709**, 022 (2007); M. Shifman and A. Yung, Phys. Rev. D **77**, 125016 (2008); J. M. Baptista, arXiv:0810.3220 [hep-th].
- [10] D. Tong, Phys. Rev. D **69**, 065003 (2004); M. Shifman and A. Yung, Phys. Rev. D **70**, 045004 (2004); A. Hanany and D. Tong, JHEP **0404**, 066 (2004); M. Eto, Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, Phys. Rev. D **72**, 025011 (2005); M. Eto *et al.*, Nucl. Phys. B **780**, 161 (2007);
- [11] K. Hashimoto and D. Tong, JCAP **0509**, 004 (2005); M. Eto, K. Hashimoto, G. Marmorini, M. Nitta, K. Ohashi and W. Vinci, Phys. Rev. Lett. **98**, 091602 (2007).
- [12] D. Tong, arXiv:hep-th/0509216. M. Eto, Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, J. Phys. A **39**, R315 (2006); M. Shifman and A. Yung, Rev. Mod. Phys. **79**, 1139 (2007); D. Tong, arXiv:0809.5060 [hep-th]; K. Konishi, arXiv:0809.1370 [hep-th].
- [13] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, Phys. Rev. Lett. **96**, 161601 (2006)
- [14] R. Auzzi, M. Eto and W. Vinci, JHEP **0711**, 090 (2007); JHEP **0802**, 100 (2008).
- [15] L. Ferretti, S. B. Gudnason and K. Konishi, Nucl. Phys. B **789**, 84 (2008); M. Eto, T. Fujimori, S. B. Gudnason, K. Konishi, M. Nitta, K. Ohashi and W. Vinci, Phys. Lett. B **669**, 98 (2008)
- [16] E. B. Bogomolny and M. S. Marinov, Yad. Fiz. **23**, 676 (1976).